Proceedings of the
International Conference on Mechanical Engineering 2011
(ICME2011) 18-20 December 2011, Dhaka, Bangladesh

ICME11-FL-050

EFFECT OF DIAMETER RATIO OF ADIABATIC CYLINDER ON NATURAL CONVECTION FROM A SQUARE OPEN CAVITY

Fariza Tul Koabra[1] **and Md. Abdul Alim**[2]

[1]Department of Mathematics, Eden Mohila College, Dhaka,Bangladesh,
[2]Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh

ABSTRACT

A numerical analysis is carried out to study the performance of natural convection inside a square open cavity. An adiabatic circular cylinder is placed at the center of the cavity and the sidewall in front of the breathing space is heated by a constant heat flux. The top and bottom walls are kept at the ambient constant temperature. Two-dimensional forms of Navier-Stokes equations along with the energy equations are solved using Galerkin finite element method. Results are obtained for a range of Grashof number from 10^3 to 10^6 at Pr = 0.71 with constant physical properties. The parametric studies for a wide range of governing parameters show consistent performance of the present numerical approach to obtain as stream functions and temperature profiles. The computational results indicate that the heat transfer coefficient is strongly affected by Grashof number. An empirical correlation is developed by using Nusselt number and Grashof number.

Keywords: Open Cavity, Natural Convection, Finite Element, Grashof Number.

1. INTRODUCTION

Natural convection in enclosures has attracted considerable interest of investigators. Such type of flow has a wide range of applications, for example, multi-pane windows, cooling of electronic equipment, solar thermal central receiver design, aircraft break-housing systems design and in other building and equipment components. Especially recently, sloped windows and skylights have been more and more frequently applied in buildings, which makes it necessary to gain more understanding on the natural convection in the cavities.

A large numbers of literatures are available which deal with the study of natural convection in enclosures [1-3] with either vertical or horizontal imposed heat flux or temperature difference. Hadjisophocleous et al. [4] solved the natural convection of a square cavity problem by no orthogonal boundary fitted coordinate system. However, they compared their results with that of de Vahl Davis [5] and Markatos and Perikleous [6] which is a regular geometry problem. Chan and Tien [7] studied numerically shallow open cavities and also made a comparison study using a square cavity in an enlarged computational domain. They found that for a square open cavity having an isothermal vertical side facing the opening and two adjoining adiabatic horizontal sides, satisfactory heat transfer results could be obtained, especially at high Rayleigh numbers. In a similar way, Mohamad [8] studied inclined open square cavities, by considering a restricted computational domain. Different from those by Chan and Tien [9], gradients of both velocity components were set to zero at the opening plane. It was found that heat transfer was not sensitive to inclination angle and the flow was unstable at high Rayleigh numbers and small inclinations angles. G. Saha et al [10] investigated a numerical simulation of two-dimensional laminar steady-state natural convection in a square tilt open cavity. The results show that the Nusselt numbers increases with the Rayleigh numbers. Also the average Nusselt number changes substantially with the inclination angle of the cavity while better thermal performance was also sensitive to the boundary condition of the heated wall.

Finite element (FE) analysis is a method to numerically solve partial differential equations which can be applied to many problems in engineering. The method has been extended to solve problems in several other fields such as in the field of heat transfer, electromagnetic, biomechanics, complex structural problems [11-13]. In spite of the great success of the method in these fields, its application to fluid mechanics is still under intensive research. This is due to the fact that the governing differential equations for general flow problems consist of several coupled equations that are inherently nonlinear. Accurate numerical solutions thus require a vast amount of computer time and data storage. One-way to minimize the amount of computer time and data storage used is to employ an adaptive meshing technique [14]. The technique places small elements in the regions of large change in the solution gradients to increase solution accuracy, and at the same time, uses large elements in the other regions to reduce the computational time and computer memory.

The objective of the present study is to investigate the effects of diameter ratio of adiabatic cylinder on natural convection placed inside an open cavity and to investigate, by developing a finite element formulation suitable for the analysis of steady-state natural convection flow problems. As the first step toward accurate flow solutions using the adaptive meshing technique. The paper starts from the Navier-Stokes equations together with the energy equation to derive the corresponding finite element equations. The computational procedure used in the development of the computer program is described. The finite element equations derived and then the computer programs developed are then evaluated by example of natural convection in a square open cavity.

2. MATHEMATICAL FORMULATION

Natural convection is governed by the differential equations expressing conservation of mass, momentum and energy. The present flow is considered steady, laminar, incompressible and two-dimensional. The viscous dissipation term in the energy equation is neglected. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature changes, and to couple in this way the temperature field to the flow field. The governing equations in non-dimensional form are written as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (1)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\sqrt{\left(\frac{1}{Gr}\right)}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\sqrt{\left(\frac{1}{Gr}\right)}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\theta \qquad (3)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr\sqrt{Gr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (4)$$

with the boundary conditions

$$U(X,0)=U(X,1)=U(0,Y)=0,$$

$$V(X,0)=V(X,1)=V(0,Y)=0,$$

$$\frac{\partial\theta}{\partial Y}(X,0)=\frac{\partial\theta}{\partial Y}(X,1)=0$$

$$\frac{\partial\theta}{\partial X}(0,Y)=-1$$

$$\theta(1,Y)=0 \text{ if } U<0 \text{ and } \frac{\partial\theta}{\partial X}(1,Y)=0 \text{ if } U>0$$

$$\frac{\partial\theta}{\partial S}(0.5d_r\cos\theta, 0.5d_r\sin\theta)=0$$

Equations (1)-(4) were normalized using the following dimensionless scales:

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{u}{U_o},\ V=\frac{v}{U_o},$$

$$P=\frac{p-p_\infty}{\rho U_o^2},\quad \theta=\frac{T-T_\infty}{\Delta T},\quad Pr=\frac{\upsilon}{\alpha},$$

$$Gr=\frac{g\beta\Delta T L^3}{\upsilon^2}$$

$$\Delta T=\frac{qL}{k}\quad d_r=\frac{D}{L}$$

Here *Pr* and *Gr* are Prandtl and Grashof number respectively. The reference velocity Uo is related to the buoyancy force term and is defined as

$$U_o=\sqrt{g\beta L\Delta t}$$

The Nusselt number (Nu) is one of the important dimensionless parameters to be computed for heat transfer analysis in natural convection flow. The local Nusselt number can be obtained from the temperature field by applying

$$Nu_x=-\frac{1}{\theta(0,Y)}$$

and the average or overall Nusselt number was calculated by integrating the temperature gradient over the heated wall as

$$Nu=-\int_0^1\frac{1}{\theta(0,Y)}dY$$

3. NUMERICAL PROCEDURE

The velocity and thermal energy equations (1)-(4) result in a set of non-linear coupled equations for which an iterative scheme is adopted. To ensure convergence of the numerical algorithm the following criteria is applied to all dependent variables over the solution domain

$$\sum\left|\phi_{ij}^{m}-\phi_{ij}^{m-1}\right|\le 10^{-5}$$

where ϕ represents a dependent variable U, V, P, and T; the indexes i, j indicate a grid point; and the index m is the current iteration at the grid level. The six node triangular element is used in this work for the development of the finite element equations. All six nodes are associated with velocities as well as temperature; only the corner nodes are associated with pressure. This means that a lower order polynomial is chosen for pressure and which is satisfied through continuity equation. The velocity component and the temperature distributions and linear interpolation for the pressure distribution according to their highest derivative orders in the differential Eqs (1)-(4) as

$$U(X,Y)=N_\alpha U_\alpha \qquad (5)$$

$$V(X,Y)=N_\alpha V_\alpha \qquad (6)$$

$$T(X,Y)=N_\alpha T_\alpha \qquad (7)$$

$$P(X,Y) = H_\lambda P_\lambda \tag{8}$$

where α = 1, 2, … …, 6; λ= 1, 2, 3; $N\alpha$ are the element interpolation functions for the velocity components and the temperature, and H_λ are the element interpolation functions for the pressure.

To derive the finite element equations, the method of weighted residuals [15] is applied to the continuity Eq. (1), the momentum Eqs (2)-(3) and the energy Eq. (4).

4. RESULTS AND DISCUSSIONS

In this problem a square open cavity with the left vertical wall is at constant heat flux, as shown in Figure 1, while the top and bottom walls are kept at the ambient constant temperature**.** The fluid concerned is air with Prandtl number 0.71. Results are obtained for a range of Grashof number from 10^3 to 10^6 at Pr = 0.71 with constant physical properties**.** The governing mass, momentum and energy equations are expressed in a normalized primitive variables formulation. Here a finite element method for steady-state incompressible natural convection flows has been developed. The streamlines and isotherms are produced, heat transfer characteristics is obtained for Grashof numbers from 10^3 to 10^6 and for d_r=0.2, 0.3 and 0.4. The results show that the Nusselt numbers increases with the Grashof numbers. Also the Nusselt number changes substantially with the diameter ratios of the cylinder of the cavity while better thermal performance is also sensitive to the boundary condition of the heated wall.

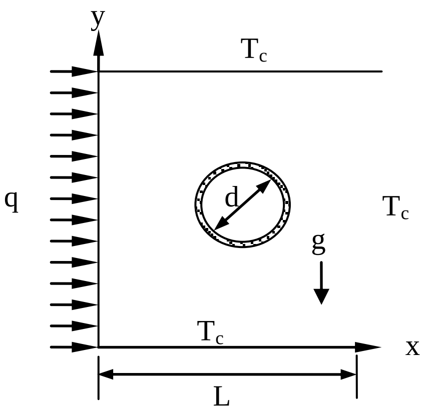


Fig 1. Schematic diagram of the square open cavity

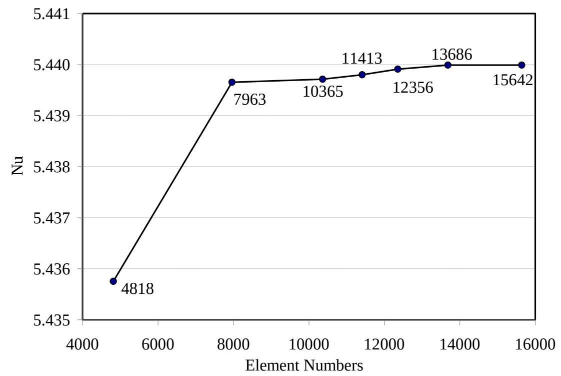


Fig 2. Convergence of average Nusselt number with grid refinement for Gr = 10^6 and d_r = 0.2

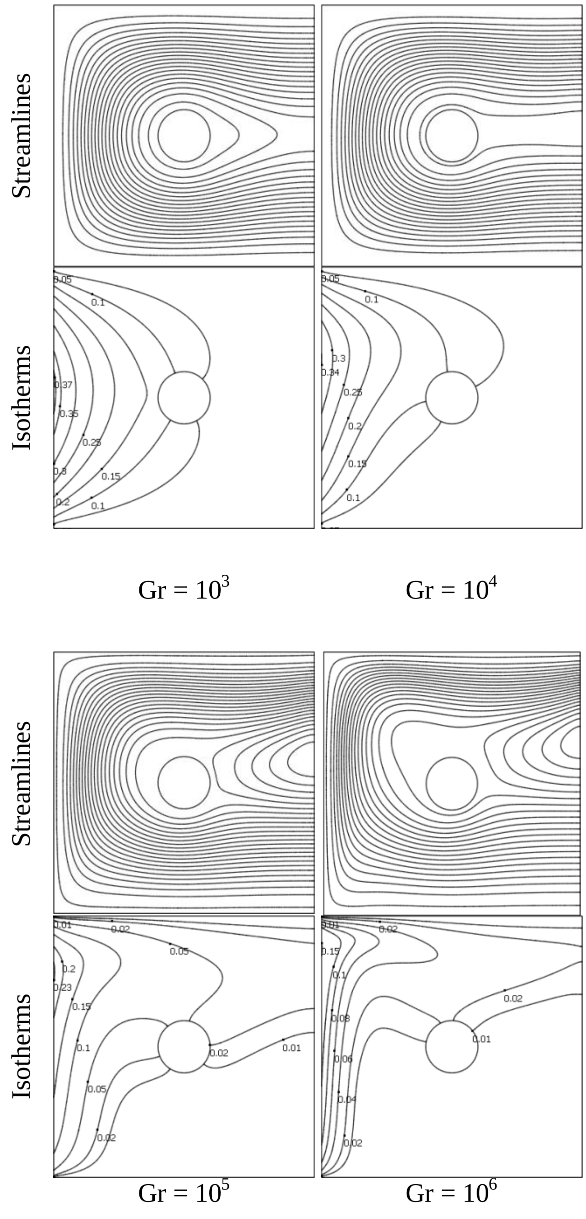


Fig 3. Streamlines and Isotherms patterns for d_r = 0.2

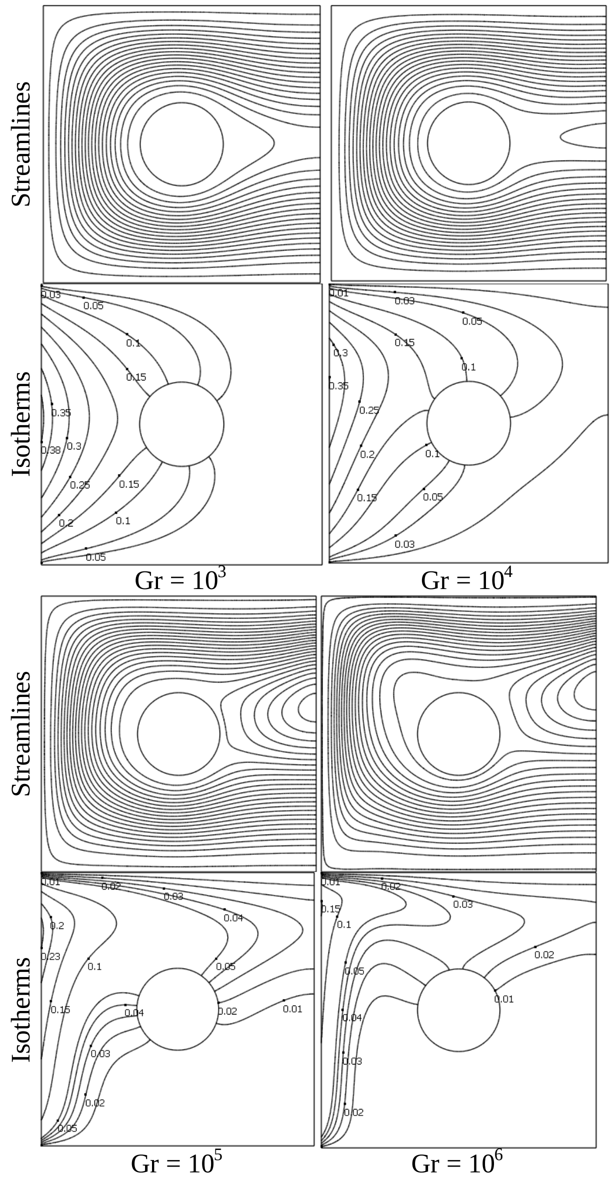
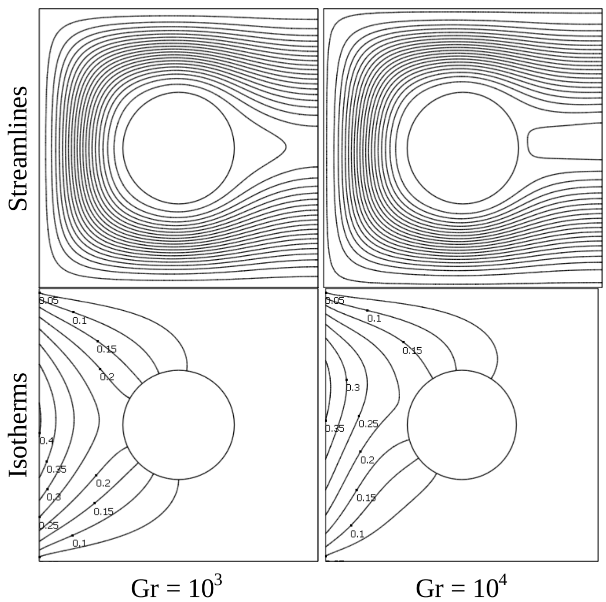


Fig 4. Streamlines and Isotherms patterns for $d_r = 0.3$



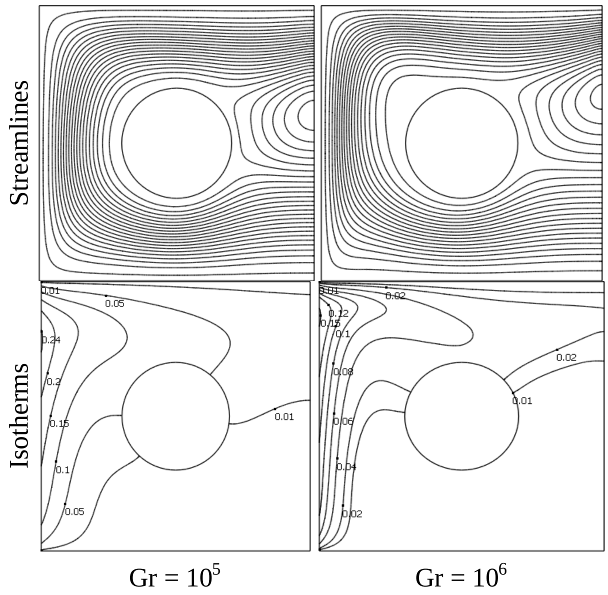


Fig 5. Streamlines and Isotherms patterns for $d_r = 0.4$

In order to obtain grid independent solution, a grid refinement study is performed for a square open cavity with $Gr = 10^6$ and $d_r = 0.2$. Figure 2 shows the convergence of the average Nusselt number, Nu at the heated surface with grid refinement. It is observed that grid independence is achieved with 13686 elements where there is insignificant change in Nu, with further increase of mesh elements. Grid independent solution is ensured by comparing the results of different grid meshes for $Gr = 10^6$, which was the highest Grashof number. The total domain is discretized into 4806 elements that results in 32643 nodes. In order to validate the numerical code, pure natural convection with Pr = 0.71 in a square open cavity was solved, and the results were compared with those reported by Hinojosa et al. [16], obtained with an extended computational domain. In Table 1, a comparison between the average Nusselt numbers is presented.

Table 1: Comparison of the results for the constant surface temperature with Hinojosa et al [16] at Pr = 0.71

Gr	Nu		
	Present work	Hinojosa et al [16]	Difference (%)
10^3	1.32	1.30	1.54
10^4	3.45	3.44	0.29
10^5	7.41	7.44	0.40
10^6	14.44	14.51	0.48

A comparison between the steady-state patterns of streamlines from Grashof numbers of 10^3 to 10^6 with different diameter ratios of the circular cylinder placed inside the cavity is presented in Figure 3 to 5. For the isotherm, the figures show that as the Grashof number and the diameter ratios increase, the buoyancy force increases and the thermal boundary layers become thinner. For the streamlines, the figures show that the fluid enters from the bottom of the aperture, circulates in a clockwise direction following the shape of the cavity, and leaves toward the upper part of the aperture. The streamline patterns is very similar for first two Grashof numbers, but the fluid moves faster for Gr = 10^4. Also, for Gr = 10^5 and 10^6, the streamline patterns is similar but the upper boundary layer becomes thinner and faster, the velocity of the air flow moving toward the aperture increases, and the area that is occupied by the leaving hot fluid decreases compared with that of the entering fluid. The results are presented in terms of streamlines and isotherm patterns. The variations of the average Nusselt number and average temperature are also highlighted.

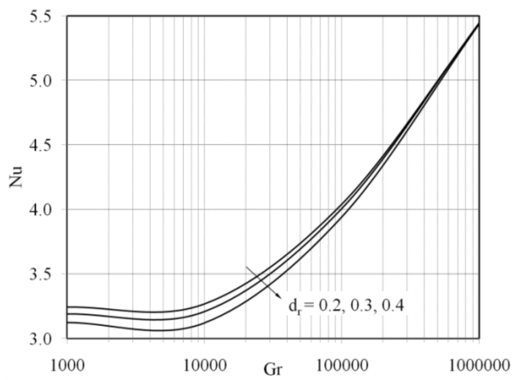


Fig 6. Average Nusselt number as a function of Grashof Number for different diameter ratio

5. CONCLUSION

A numerical investigation on natural convection around an adiabatic circular cylinder in an open enclosure has been performed by using finite element method. The results show that the heat transfer rate increases with the decrease of heat source size and increase of Grashof number. The maximum non-dimensional temperature also decreases as Gr increases keeping any of these parameters constant.

6. NOMENCLATURE

Symbol	Meaning	Unit
g	gravitational acceleration	ms^{-2}
Gr	Grashof number	
k	thermal conductivity of the fluid	$Wm^{-1}K^{-1}$
L	hight and width of the enclosure	m
Nu	Nusselt number	
p	pressure	Nm^{-2}
P	non-dimensional pressure	
Pr	Prandtl number	
q	heat flux	Wm^{-2}
T	dimensional temperature	
T	temperature	K
u, v	velocity components	ms^{-1}
U, V	non-dimensional velocity components	
x, y	Cartesian coordinates	m
X, Y	non-dimensional Cartesian coordinates	
α	thermal diffusivity	m^2s^{-1}
β	thermal expansion coefficient	K^{-1}
ρ	density of the fluid	kgm^{-3}
υ	kinematic viscosity of the fluid	m^2s^{-1}
θ	non-dimensional temperature	

7. REFERENCES

[1] Ganzarolli, M.M., Milanez, L.F. (1995). Natural convection in rectangular enclosures heated from below and symmetrically cooled from the sides, Int. J. Heat Mass Transfer, vol. 38, 1063–1073.
[2] Coffin D. W. and Bloom F., 1999. Elastica solution for the hygrothermal buckling of a beam. Int. J. Nonlinear Mech., 34, 935–947.
[3] Dalal, A., Das, M.K. (2005). Laminar natural convection in an inclined complicated cavity with spatially variable wall temperature, Int. J. Heat Mass Transfer, vol. 48, 3833–3854.
[4] Hadjisophocleous, G.V., Sousa, A.C.M., Venart, J.E.S. (1988). Prediction of transient natural convection in enclosures of arbitrary geometry using a nonorthogonal numerical model, Numer. Heat Transfer, vol. 13, 373–392.
[5] Vahl Davis, G. de. (1983). Natural convection of air in a square cavity: a benchmark numerical solution, Int. J. Numer. Methods Fluids, vol. 3, 249–264.
[6] Markatos, N. C, Perikleous, K.A. (1984). Laminar and turbulent natural convection in an enclosed cavity, Int. J. Heat Mass Transfer, vol. 27, 755–772.
[7] Chan, Y. L. and Tien, C. L. (1985). A Numerical study of two-dimensional laminar natural convection in a shallow open cavity, Int. J. Heat Mass Transfer, vol. 28, 603-612.
[8] Mohamad, A. (1995). Natural convection in open cavities and slots, Numerical Heat Transfer A, vol. 27, 705-716.
[9] Chan, Y. L. and Tien, C. L. (1985). A Numerical study of two-dimensional natural convection in square open cavities, Numerical Heat Transfer, vol. 8, 65-80.
[10] Saha G, Saha S and Mamun A. H. (2007). A finite element method for steady-state natural convection in a square tilt open cavity, ARPN Journal of Engineering and Applied Science, vol. 2, 41-49
[11] Dechaumphai, P. (1999). Finite Element Method in Engineering, second Ed., Chulalongkorn University Press, Bangkok.
[12] Jini, J. (1993). The finite element method in electromagnetics, John Wiley & Sons, New York.
[13] Gallagher, R. H., Simon, B. R., Johnson, P. C. and Gross, J. F. (1982). Finite elements in biomechanics, John Wiley & Sons, New York.
[14] Dechaumphai, P. (1995). Adaptive finite element technique for heat transfer problems, Energy, Heat & Mass transfer, vol. 17, 87-94.
[15] Zienkiewicz, O. C. and Taylor, R. L. 1991. The finite element method, Fourth Ed., McGraw-Hill.
[16] Hinojosa, J. F., Estrada, C. A., Cabanillas, R. E., and Alvarez, G. 2005. Nusselt number for the natural convection and surface thermal radiation in a square tilted open cavity, Int. Com. In Hear and Mass Transfer, vol. 32, 1184-1192.